

Energy-efficient Resource Allocation for Uplink OFDMA Systems Using Correlated Equilibrium

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Abstract—In this work, we propose a correlated equilibrium (CE)-based energy-efficient resource allocation scheme for uplink OFDMA systems. At first, we construct an energy-efficient resource allocation game, where each subcarrier is viewed as a player to choose the most satisfying user, and the objective is to balance the tradeoff between the total energy efficiency and the fairness. Since the CE can achieve better performance by helping the non-cooperative players coordinate their strategies, we employ the CE to analyze the proposed game. Next, we derive the condition under which the CE is Pareto optimal and employ linear programming duality to show its closed-form expressions. Furthermore, we present a linear programming method and a distributed algorithm based on the regret matching procedure to implement the CE, respectively. Simulation results demonstrate that our scheme is able to achieve good convergence, Pareto optimality, and fairness.

Index Terms—OFDMA, correlated equilibrium, regret matching, linear programming, energy efficiency, resource allocation.

I. INTRODUCTION

OFDMA becomes a promising multiple access technique for next-generation networks. By efficient resource allocation, OFDMA can offer multiuser diversity gain to improve system performance greatly. Considering the distributed and individualistic behaviors of the users, i.e., the users' different power constraints, a resource allocation scheme is needed in a distributed, low-complexity fashion. Hence, game theory is assumed to be one of the most suitable methods to model this scenario. A line of literature on game-theoretical approaches provides several helpful solutions to resolve the following aspects:

- 1) Green communications attract more and more attention, and hence, the energy efficiency is becoming increasingly important, and should be treated as a performance metric. Specifically, the authors of [1], [2] address link adaptive transmission for maximizing energy efficiency.
- 2) Users close to the base station (BS) may be allocated most of the resources, whereas edge users usually suffer from starvation. To achieve fair resource allocation, autonomous auctions among the users and the Nash bargaining solutions are introduced in [3], [4].
- 3) The outcomes of individual optimization might not always be as good as those of system optimization.

The additional cooperation or coordination can improve the individual outcomes. For this observation, a virtual referee is introduced in [5].

Unfortunately, most of the existing literature consider only some of the above mentioned aspects, while neglecting the others. In order to further improve the energy efficiency, the fairness provisioning, and the mutual coordination, the above aspects are jointly considered in this work. Specifically, we propose a correlated equilibrium (CE)-based energy-efficient resource allocation scheme for uplink OFDMA systems. In particular, the CE can directly consider the ability of players to coordinate actions, hence, it spurs achieving a better solution compared to the non-cooperative Nash equilibrium (NE). Also, it is naturally attractive for distributed learning algorithms to solve discrete problems. Several wireless networking problems have been solved by using the CE (e.g., [6], [7]). Due to the specificity of the uplink OFDMA resource allocation, it is difficult to directly apply the aforementioned results to this problem. The highlights of this work are summarized as follows:

- We model the uplink resource allocation problem as a non-cooperative game with the goal of maximizing the total energy efficiency and achieving a certain fairness. In order to guarantee the stability of strategy space and the uniqueness of subcarrier allocation, we view each subcarrier as a player. Moreover, the joint user and transmit power selection is simplified as the user selection which is characterized by a discrete strategy space.
- We adopt the CE concept to analyze the outcome of the proposed game, and thus, the subcarriers are aware of coordinating the resource usage. Moreover, we derive the condition under which the CE of the proposed game is Pareto optimal and employ linear programming duality to show its closed-form expressions. Meanwhile, we present two approaches to implement the CE, namely a linear programming method and a distributed algorithm based on the regret matching procedure. The resulting CE can help us achieve the energy-efficient resource allocation.

II. SYSTEM MODEL AND PROBLEM FORMULATION

Consider the uplink transmission in an OFDMA system where a BS and N users located in the same cell. Let

$\mathcal{N} = \{1, \dots, n, \dots, N\}$ be a set of all users. Each user transmits the data over a subset of subcarriers among all available subcarriers in a set $\mathcal{K} = \{1, \dots, k, \dots, K\}$. Then, the throughput of user n on subcarrier k is given by

$$T_{n,k} = \log(1 + p_{n,k} \gamma_{n,k}), \quad (1)$$

where $\log(\cdot)$ is the natural logarithm, and $p_{n,k}$ and $\gamma_{n,k}$ are the transmit power and the channel signal-to-noise ratio (SNR) of user n on subcarrier k , respectively. More precisely, $\gamma_{n,k}$ is defined as $\gamma_{n,k} = \xi_n H_{n,k} / \sigma^2$, where $H_{n,k}$ is the channel gain between the BS and user n over subcarrier k in the context of slowly time varying and frequency-selective propagation channel, σ^2 is the noise power and is identical for all links, and ξ_n is an SNR gap function. To this end, the throughput of user n can be obtained by

$$T_n = \sum_{k \in \mathcal{K}_n} T_{n,k}, \quad (2)$$

where \mathcal{K}_n denotes a set of subcarriers allocated to user n . Due to the exclusive subcarrier assignment for uplink systems, we have $\mathcal{K}_n \cap \mathcal{K}_{\tilde{n}} = \emptyset, \forall n \neq \tilde{n}, \tilde{n} \in \mathcal{N}$, and $\sum_{n \in \mathcal{N}} \mathcal{K}_n \subseteq \mathcal{K}$.

For the energy-efficient transmission mode, taking user n for example, power consumption should contain transmit power (denoted by p_n) and circuit power (denoted by p_c) [1], [2]. Then, its overall power consumption is written as

$$p_n^{total} = \frac{\sum_{k \in \mathcal{K}_n} p_{n,k}}{\beta} + p_c = \frac{p_n}{\beta} + p_c, \quad (3)$$

where $\beta \in [0, 1]$ is the power amplifier efficiency.

The goal of energy-efficient communications is to supply what the users most want at the lowest possible power cost. That is, with energy Δe consumed in a duration Δt , it is desirable for user n to maximize

$$\frac{T_n \Delta t}{\Delta e}, \quad (4)$$

which is equivalent to maximizing

$$\frac{T_n}{\Delta e / \Delta t} = \frac{T_n}{p_n^{total}}. \quad (5)$$

Here, Eq. (5) represents the number of bits transmitted per Joule of energy of user n , and it will be treated as a measure of energy efficiency of user n in what follows.

III. ENERGY-EFFICIENT RESOURCE ALLOCATION GAME AND CORRELATED EQUILIBRIUM

A. Design

We focus on the resource allocation in terms of subcarrier and transmit power, from a non-cooperative game theoretic perspective. Usually, a user serves as a player to choose its transmit power strategy in continuous domain. If the game takes into account subcarrier allocation, the above setup is unusable. That is because we may face some awkward dilemmas, such as the changing strategy space, the contradiction with the exclusive subcarrier assignment, and the confliction between different strategy spaces. In order to solve these problems, we employ all subcarriers as the players and integrate the

joint user and transmit power selection into the user selection which is characterized by discrete strategy space. Formally, the energy efficient resource allocation game is denoted as $G_{EERA} = \langle \mathcal{K}, \{\mathcal{A}_k\}_{k \in \mathcal{K}}, \{u_k\}_{k \in \mathcal{K}} \rangle$, where the components of G_{EERA} are given in the list:

1) \mathcal{K} is regarded as a set of players.

2) \mathcal{A}_k is a finite set of strategies of subcarrier k , i.e., $\mathcal{A}_k = \mathcal{N}, \forall k \in \mathcal{K}$. Once the subcarrier chooses its own user, the transmit power strategy can be determined, as will be described later in detail. Hence, \mathcal{A}_k implies the joint set of user and transmit power selection strategies. Also, the space for the strategy profiles is $\mathcal{A} = \mathcal{A}_1 \times \dots \times \mathcal{A}_K$.

3) u_k is a individual utility function. Note that the logarithmic utility function is associated with the proportional fairness for the utility-based optimization [4]. In order to show the improving degree of total energy efficiency and the fairness provisioning, it can be expressed as

$$u_k(A_k, A_{-k}) = \sum_{n=1}^N \alpha_n \log \left(\frac{T_n(A_k, A_{-k})}{p_n^{total}(A_k, A_{-k})} - E_{n,\text{thr}} \right), \quad (6)$$

where α_n is the weight factor of user n ' energy efficiency, $E_{n,\text{thr}}$ is the threshold of energy efficiency of user n , $A_k \in \mathcal{A}_k$ is the strategy of player k , and A_{-k} represents the joint strategies of the other players.

Since each player tries to maximize its own utility, G_{EERA} can be formulated as a set of optimization problems

$$\max_{A_k \in \mathcal{A}_k} u_k(A_k, A_{-k}), \quad \forall k \in \mathcal{K}, \quad (7)$$

subject to

$$p_n(A) \leq p_{n,\text{max}}, \quad \forall n \in \mathcal{N}, \quad (7a)$$

where $A = (A_k, A_{-k})$ is called a strategy profile and $p_{n,\text{max}}$ is the maximum transmit power of user n . The constraints on maximum transmit power for individual users are specified in (7a). Note that u_k is also a function of A_{-k} , because it depends on the throughput and overall power consumption of each user which are related to A_{-k} .

B. Analysis

In order to analyze the outcome of G_{EERA} , we concentrate on the correlated equilibrium (CE). The CE permits to coordinate the user selection strategies among the subcarriers, hence, can bridge the gap between the individual optimization and the system optimization.

Definition 1 [8]: Let $\Delta \mathcal{A}$ be a set of probability distributions on \mathcal{A} . For G_{EERA} , a correlated strategy $P = (P(A))_{A \in \mathcal{A}} \in \Delta \mathcal{A}$ is a CE if for every strategy A_k such that $P(A_k) > 0$, and other strategy $\tilde{A}_k \in \mathcal{A}_k$, it holds that

$$\sum_{A_{-k}} P(A_k, A_{-k}) \left[u_k(A_k, A_{-k}) - u_k(\tilde{A}_k, A_{-k}) \right] \geq 0. \quad (8)$$

P provides each subcarrier k with a private "recommendation" $A_k \in \mathcal{A}_k$, so as to allow a weak form of cooperation.

Theorem 1: For the energy-efficient resource allocation game G_{EERA} , a CE always exists.

Proof: We observe from Eq. (8) that the set of correlated equilibria is nonempty in every finite game [8]. Hence, it is justified, and enables the application of G_{EERA} . ■

Furthermore, we characterize the set of all Pareto optimal correlated strategies. A correlated strategy P is Pareto optimal if there does not exist another correlated strategy \tilde{P} such that $\sum_{A \in \mathcal{A}} \tilde{P}(A) u_k(A) \geq \sum_{A \in \mathcal{A}} P(A) u_k(A)$ for $\forall k \in \mathcal{K}$ with at least one inequality strict.

Lemma 1: The problem of checking whether or not G_{EERA} has a Pareto optimal correlated strategy reduces to the problem of computing a CE that maximizes the sum of the players' expected utilities.

Proof: Given G_{EERA} , the Pareto optimal correlated strategy can lead each player to earn the maximum-possible utility, and let the sum of these utilities be u . If G_{EERA} has a Pareto optimal correlated strategy, this outcome is also a correlated equilibrium with total utility u . If there is no Pareto optimal correlated strategy, there is no outcome with total utility at least u , and hence no distribution over outcomes (CE or otherwise) with total expected utility at least u . ■

Therefore, we can obtain the CE mentioned above by solving the following optimization problem

$$\max_P \sum_{A \in \mathcal{A}} P(A) \sum_{k \in \mathcal{K}} u_k(A), \quad (9)$$

subject to constraint (8), and

$$\sum_{A \in \mathcal{A}} P(A) = 1. \quad (9a)$$

$$P(A) \geq 0 \quad \forall A \in \mathcal{A}. \quad (9b)$$

Then, the dual problem of (9) can be shown in the form of matrices:

$$\min w_2, \quad (10)$$

$$\mathbf{w}_1 \mathbf{A}_1 - w_2 \mathbf{A}_2 \leq -\mathbf{u}, \quad (10a)$$

$$\mathbf{w}_1 \geq \mathbf{0}, \quad (10b)$$

where \mathbf{w}_1 is the row vector which consists of $KN(N-1)$ variables, and w_2 is also a variable. Assume that the strategy profile A is in the i^{th} position of \mathcal{A} . \mathbf{A}_1 is a $KN(N-1) \times |\mathcal{A}|$ matrix where the entries are determined by the coefficients of constraint (8), and the i^{th} column corresponds to the strategy profile A , $\mathbf{A}_2 = [1, \dots, 1]$ is a $|\mathcal{A}|$ dimensional row vector, and \mathbf{u} is a $|\mathcal{A}|$ dimensional row vector with the i^{th} entry $\sum_{k \in \mathcal{K}} u_k(A)$. Moreover, the dual problem can be obtained by the linear programming method.

IV. IMPLEMENTATION OF CORRELATED EQUILIBRIUM

A. Linear Programming Method

We take a two-subcarrier-two-user game as an example to obtain the CE by using the linear programming method. That is, $\mathcal{K} = \{1, 2\}$ is the set of players, and $\mathcal{A}_1 = \mathcal{A}_2 = \{1, 2\}$ is the set of strategies. A general utility is shown in Table I, in which subcarrier 1 is the row player, subcarrier 2 is the column player, and $u_k(i, j)$ is the utility for subcarrier k when the strategy profile is in the i^{th} row and the j^{th} column. Let $P = (P(1, 1), P(2, 2), P(2, 1), P(1, 2))$ be a correlated strategy. According to Definition 1, we can specify the CE by means of linear constraints as follows:

$$P(1, 1)(u_1(1, 1) - u_1(2, 1)) \geq P(1, 2)(u_1(2, 2) - u_1(1, 2)), \quad (12a)$$

TABLE I
UTILITY OF TWO-SUBCARRIER-TWO-USER G_{EERA}

	1	2
1	$u_1(1, 1), u_2(1, 1)$	$u_1(1, 2), u_2(1, 2)$
2	$u_1(2, 1), u_2(2, 1)$	$u_1(2, 2), u_2(2, 2)$

TABLE II
THE SET OF CORRELATED EQUILIBRIA OF G_{EERA}^1

P	$P(1, 1)$	$P(2, 2)$	$P(2, 1)$	$P(1, 2)$
CE1	1	0	0	0
CE2	0	1	0	0
CE3	$\frac{1}{(1+\alpha)(1+\beta)}$	$\frac{\alpha\beta}{(1+\alpha)(1+\beta)}$	$\frac{\beta}{(1+\alpha)(1+\beta)}$	$\frac{\alpha}{(1+\alpha)(1+\beta)}$
CE4	$\frac{1}{1+\beta+\alpha\beta}$	$\frac{\alpha\beta}{1+\beta+\alpha\beta}$	$\frac{\beta}{1+\beta+\alpha\beta}$	0
CE5	$\frac{1}{1+\alpha+\alpha\beta}$	$\frac{\alpha\beta}{1+\alpha+\alpha\beta}$	0	$\frac{\alpha}{1+\alpha+\alpha\beta}$

$$P(2, 2)(u_1(2, 2) - u_1(1, 2)) \geq P(2, 1)(u_1(1, 1) - u_1(2, 1)), \quad (12b)$$

$$P(1, 1)(u_2(1, 1) - u_2(1, 2)) \geq P(2, 1)(u_2(2, 2) - u_2(2, 1)), \quad (12c)$$

$$P(2, 2)(u_2(2, 2) - u_2(2, 1)) \geq P(1, 2)(u_2(1, 1) - u_2(1, 2)), \quad (12d)$$

$$P(1, 1) \geq 0, P(1, 2) \geq 0, P(2, 1) \geq 0, P(2, 2) \geq 0, \quad (12e)$$

$$P(1, 1) + P(1, 2) + P(2, 1) + P(2, 2) = 1, \quad (12f)$$

where constraints (12a)-(12d) represent the constraints of CE in (8) and the others guarantee that P is a valid probability distribution. Moreover, we relax constraint (12f) and formulate another problem by adding an objective function to it:

$$\min_P -(P(1, 1) + P(1, 2) + P(2, 1) + P(2, 2)), \quad (13)$$

subject to constraints (12a) to (12e).

Let $\alpha = |u_1(1, 1) - u_1(2, 1)| / |u_1(2, 2) - u_1(1, 2)|$ and $\beta = |u_2(1, 1) - u_2(1, 2)| / |u_2(2, 2) - u_2(2, 1)|$. Similar to [9], we can take advantage of the linear programming method to derive the set of correlated equilibria as follows:

- *Case 1:* The game without dominated strategies, i.e., $u_1(1, 1) \neq u_1(2, 1)$, $u_1(2, 2) \neq u_1(1, 2)$, $u_2(1, 1) \neq u_2(1, 2)$, and $u_2(2, 2) \neq u_2(2, 1)$:
 - 1) If and only if $u_1(1, 1) > u_1(2, 1)$, $u_1(2, 2) > u_1(1, 2)$, $u_2(1, 1) > u_2(1, 2)$, and $u_2(2, 2) > u_2(2, 1)$, the game is denoted by G_{EERA}^1 . Then, the set of correlated equilibria of G_{EERA}^1 is a polytope of $\Delta\mathcal{A}$ with the five vertices which are shown in Table II.
 - 2) If and only if $u_1(1, 1) < u_1(2, 1)$, $u_1(2, 2) < u_1(1, 2)$, $u_2(1, 1) < u_2(1, 2)$, and $u_2(2, 2) < u_2(2, 1)$, the game is denoted by G_{EERA}^2 . If we obtain a CE of G_{EERA}^1 with parameters α and β , namely $P = (P(1, 1), P(2, 2), P(2, 1), P(1, 2))$, the correlated strategy denoted by $P = (P(2, 1), P(1, 2), P(1, 1), P(2, 2))$ is a CE of G_{EERA}^2 with parameters α and $1/\beta$.
 - 3) Otherwise, the game is denoted by G_{EERA}^3 . Then, the sets of correlated equilibria and Nash equilibria of G_{EERA}^3 coincide. Moreover, the set of Nash equilibria is restricted to the mixed strategy: play 1 with probability $1/(1+\beta)$ for subcarrier 1 and $1/(1+\alpha)$ for subcarrier 2.

- *Case 2:* The game with weakly dominated strategies, i.e., either $\alpha = 0$, or $\beta = 0$ or both: the set of correlated equilibria corresponds to the convex hull of the vertices in Table II.
- *Case 3:* Otherwise, the game with strictly dominated strategies: the set of correlated equilibria coincides with the set of Nash equilibria of the proposed game.

For a multi-subcarrier multiuser case, the linear programming method can still be employed to obtain the CE. However, all information is required to be available for optimization, which is inappropriate for the distributed structure.

B. Distributed Learning Algorithm

In this subsection, we present a distributed learning algorithm based on the regret matching procedure of [8] to obtain the CE. Given a history of play (i.e., $(A^\tau)_{\tau=1}^t \in \prod_{\tau=1}^t \mathcal{A}$) and a strategy played at time t for subcarrier k (i.e., $A_k^t = n$), the average regret at time t for not having played, every time that n was played in the past, the different strategy $\tilde{n} \in \mathcal{A}_k$ is

$$R_k^t(n, \tilde{n}) = \max \{D_k^t(n, \tilde{n}), 0\}, \quad (14)$$

where $D_k^t(n, \tilde{n})$ represents the difference in subcarrier k 's average utility up to time t , if subcarrier k replaces strategy n every time in the past with the different strategy \tilde{n} , i.e.,

$$D_k^t(n, \tilde{n}) = \frac{1}{t} \sum_{\tau \leq t: A_k^\tau = n} [u_k(\tilde{n}, A_{-k}^\tau) - u_k(A^\tau)]. \quad (15)$$

Then, the energy-efficient resource allocation algorithm is executed independently by each subcarrier and summarized as follows.

1) *Initialization:* At $t = 1$, each subcarrier k initializes its strategy $A_k^1 \in \mathcal{A}_k$ arbitrarily. Its utility $u_k(A^1)$ is determined according to Eq. (6), where the transmit power is subdivided equally among all the subcarriers allocated to that user.

2) *Average Regret Value Update:* At $t + 1$, according to the history of play and utilities, subcarrier k obtains its average regret value at t , $R_k^t(A_k^t, \tilde{n})$, $\tilde{n} \neq A_k^t$, by calculating Eq. (14).

3) *Strategy Update:* Calculate the probability distribution $q_k^{t+1} \in \Delta \mathcal{A}_k$ at time $t + 1$, according to

$$q_k^{t+1}(\tilde{n}) = \begin{cases} \frac{1}{\mu} R_k^t(A_k^t, \tilde{n}), & \tilde{n} \neq A_k^t, \\ 1 - \sum_{\hat{n} \in \mathcal{A}_k: \hat{n} \neq A_k^t} \frac{1}{\mu} R_k^t(A_k^t, \hat{n}), & \tilde{n} = A_k^t. \end{cases} \quad (16)$$

Then, subcarrier k updates its strategy for time $t + 1$:

$$A_k^{t+1} = \arg \max_{\tilde{n} \in \mathcal{A}_k} q_k^{t+1}(\tilde{n}). \quad (17)$$

4) *Convergence Judgement:* Subcarrier k 's utility $u_k(A^{t+1})$ is updated according to Eq. (6), where the transmit power is subdivided equally among all the subcarriers allocated to that user. If the state of $t + 1$ is equal to that of t , stop the algorithm. Otherwise, replace $t + 1$ with t , and go to 2).

In what follows, we make some discussions on the proposed algorithm:

1) *Interpretation for Distributed Algorithm:* The subcarrier is viewed as a player, and can be implemented in any user. For example, user n which subcarrier k belongs to at time

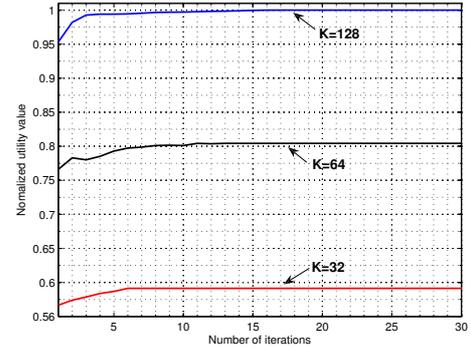


Fig. 1. Convergence behavior of the proposed algorithm.

t becomes a spokesman for subcarrier k until its strategy for time $t + 1$ is updated. The advantage is that user n can keep record of $D_k^{t-1}(n, \tilde{n})$, and we only need to calculate $u_k(\tilde{n}, A_{-k}^t) - u_k(A^t)$. In addition, each player does not need to know the individual strategies of other players or the global CSI, but only needs to know the effect of other players on its own utility function. This accords with the distributed nature.

2) *Interpretation for Transmit Power Strategy:* We adopt the equal power allocation. That is because the achieved gains are negligible compared to the increase in complexity when optimal power allocation is performed [10].

3) *Interpretation of μ in Eq. (16):* The convergence of the algorithm holds for any $\mu > 2M_k(N - 1)$ for $\forall k \in \mathcal{K}$, where M_k is an upper bound for $|u_k|$.

4) *Computational Complexity Analysis:* At each iteration, each player performs one table lookup to calculate its utility, two additions and two multiplications to update its regret value, and one random number, one multiplication and one comparison to calculate the next strategy.

5) *Convergence Analysis:* Define the relative frequency of strategy profile A played till time t as $z^t(A) = \frac{1}{t} |\tau \leq t: A^\tau = A|$. Hence, we have $z^t = (z^t(A))_{A \in \mathcal{A}} \in \Delta \mathcal{A}$. Similar to [8], we can prove that if every subcarrier follows the proposed algorithm, z^t converge almost surely as $t \rightarrow \infty$ to the set of correlated equilibria of G_{EERA} . Due to the limited space, we do not repeat it here.

V. SIMULATION RESULTS AND ANALYSIS

For a uplink OFDMA system, the weighting factors of all users are set as one, but each user has different maximum transmit power. The channel gain between the BS and its user n over subcarrier k is defined as $H_{n,k} = \kappa d_n^{-\delta} |h_{n,k}|^2$, where κ is set to 1 which depends on the propagation environment, d_n is the distance between the BS and user n , δ represents a path loss exponent, and is set to 3, and $h_{n,k} \sim \mathcal{CN}(0, 1)$ is a unitary power, Rayleigh fading coefficient. The Gaussian noise variance σ^2 is 10^{-12}W .

Fig. 1 plots the evolution of normalized utility value through the proposed algorithm, where the OFDMA systems with 16 users employing 32, 64, and 128 subcarriers are considered, respectively. The normalized utility is updated at each iteration.

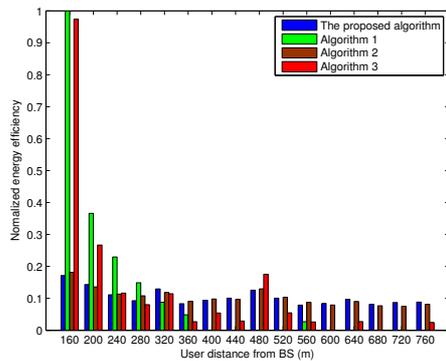


Fig. 2. Normalized energy efficiency achieved by each user as a function of the distance from the BS.

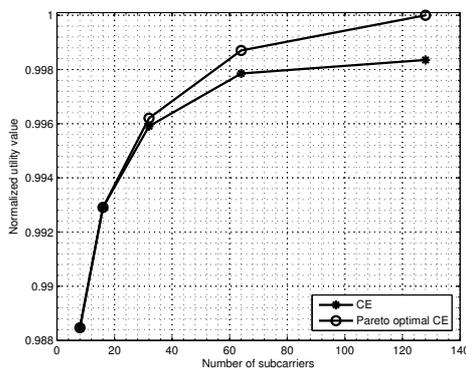


Fig. 3. Comparison of normalized utility value.

The convergence takes no more than 15 iterations with these changes. The more the subcarriers, the slower the convergence speed. That is because the increase in subcarrier numbers can result in the growth of opportunity of interaction among the players. Just for the record, similar simulation results can be achieved when more subcarriers are employed.

For comparison purpose, the following algorithms are considered as benchmarks: i) Algorithm 1: Each subcarrier is assigned to the user with the best channel gain in turn, ii) Algorithm 2: Each user choose the best subcarrier in terms of the channel gain in turn, iii) Algorithm 3: The metric of the regret matching procedure is modified as the sum of the individual energy efficiency. Also, we perform equal power allocation for all the algorithms. Fig. 2 shows the normalized energy efficiency as the function of the distance when sixteen users are considered. For Algorithm 1, most resources are allocated to the users close to the BS whereas users far from BS suffer from starvation. Algorithm 3 has very close performance to Algorithm 1 for users having good channel gains. However, it tries to prevent the excessive concentration of subcarriers, thus providing more resources for edge users. Algorithm 2 ensures almost maximum fairness at the expense of achieving low total energy efficiency. The proposed algorithm has similar behavior to Algorithm 2, and brings about slightly more energy efficiency than Algorithm 2.

In Fig. 3, we compare the normalized utility value under the CE and the Pareto optimal CE strategy, where the CE is obtained by the proposed algorithm, and the Pareto optimal CE is calculated according to Section III(B). We have $N = 8$, and $K = 8, 16, 32, 64$, and 128 , respectively. When the number of subcarriers is small ($K \leq 32$), the CE has almost the same performance as the Pareto optimal CE. When the number of subcarriers is large, the result of the Pareto optimal CE only has a slight better performance than that of the CE. For example, when $K = 128$, their performance difference is about 0.2%. This is because the competition becomes drastic and complex with an increase in the number of players, and the efficient cooperation seems particularly important.

VI. CONCLUSIONS

In this work, we design a CE-based resource allocation scheme to achieve the energy efficiency and the fairness for uplink OFDMA systems. Specifically, we model an energy-efficient resource allocation game and focus on the CE to analyze the proposed game. A linear programming method and a distributed algorithm based on the regret matching procedure are proposed to obtain the CE, respectively. From the resulting CE, we can determine the resource allocation strategies. Both the theoretical analysis and simulation results demonstrate the efficiency of the proposed scheme.

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REFERENCES

- [1] G. Miao, N. Himayat, Y. Li, and S. Talwar, "Low-complexity energy-efficient OFDMA," in Proc. of *IEEE ICC*, 2009.
- [2] G. Miao, N. Himayat, and Y. Li, "Energy-efficient link adaptation in frequency-selective channels," *IEEE Transactions on Communications*, vol. 58, no. 2, pp. 545-554, 2010.
- [3] K. Yang, N. Prasad, and X. Wang, "An auction approach to resource allocation in uplink OFDMA systems," *IEEE Transactions on Signal Processing*, vol. 57, no. 11, pp. 4482-4496, 2009.
- [4] Z. Han, Z. Ji, and K. J. R. Liu, "Fair multiuser channel allocation for OFDMA networks using Nash bargaining solutions and coalitions," *IEEE Transactions on Communications* vol. 53, no. 8, pp. 1366-1376, 2005.
- [5] Z. Han, Z. Ji, and K. J. R. Liu, "Non-cooperative resource competition game by virtual referee in multi-cell OFDMA networks," *IEEE Journal on Selected Areas on Communications*, vol. 25, no. 6, pp. 1079-1090, 2007.
- [6] M. Maskery, V. Krishnamurthy, and Q. Zhao, "Decentralized dynamic spectrum access for cognitive radios: cooperative design of a non-cooperative game," *IEEE Transactions on Communications*, vol. 57, no. 2, pp. 459-469, 2009.
- [7] V. Krishnamurthy, M. Maskery, and G. Yin, "Decentralized adaptive filtering algorithms for sensor activation in an unattended ground sensor network," *IEEE Transactions on Signal Processing*, vol. 56, no. 12, pp. 6086-6101, 2008.
- [8] S. Hart and A. Mas-Colell, "A simple adaptive procedure leading to correlated equilibrium," *Econometrica*, vol. 68, no. 5, pp. 1127-1150, 2000.
- [9] A. Armengol, "The set of correlated equilibria of 2x2 Games," [Online] Available: <http://selene.uab.es/acalvo/correlated.pdf>.
- [10] J. Lim, H. G. Myung, K. Oh, and D. J. Goodman, "Channel-Dependent Scheduling of Uplink Single Carrier FDMA Systems," in Proc. of *IEEE VTC Fall*, 2006.