Relay Power Allocation in Auction-based Game Approach

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***Abstract*— In this work, with respect to the uncertainty about the individual information, we investigate the relay power allocation problem from the energy-efficient, Pareto optimal, and competitive fairness perspective. At first, we design an easy-implementation energy efficiency metric, which aims at striking a balance between the QoS provisioning and the energy consumption. Then, an auction mechanism is proposed for relay power allocation. By transferring the auction mechanism into a game, we prove the existence, uniqueness, and Pareto optimality of the Nash equilibrium (NE) for our auction game, and show that the allocation strategies from the NE can achieve the energy efficiency in terms of the proposed metric. Next, we develop a distributed relay power allocation algorithm based on our best-response functions to reach the Pareto optimal NE. Importantly, we not only certify the convergence of the proposed algorithm, but also provide quantitative analysis on it. Extensive simulations results are conducted to confirm the validity of the analytical results.**

***Index Terms*— Wireless cooperative networks, auction theory, energy efficiency, power allocation, outage probability.**

I. Introduction

A. Motivation and Background

DUE to the global aspiration for energy saving and reduction of carbon footprint, the energy-efficient communication is becoming a crucial trend [1]. In particular, cooperative communication technique has been widely considered as a promising approach to achieve energy efficiency [2]. In order to take full advantage of the energy saving potential, some issues deserve further consideration:

• Energy efficiency metrics play an important role in comparing and assessing the energy consumption of various components and overall networks. Since future wireless networks will support a variety of services with heterogeneous QoS requirements, it is expected to seek a refined energy efficiency metric to provide a comprehensive evaluation of energy savings and general QoS performance.

• Resource allocation and coordination among the cooperative members become an important guarantee for realizing the energy-efficient advantages. Given a fixed relay, the paramount issue is how to distribute the energy resource of the relay to the nodes in need, even in a distributed and incomplete-information manner. For a multiple node-pairs relay network model, where each relay is delegated to assist one or more source-destination pairs, multiple source-destination pairs share radio resources from the same relay, and thus, efficient resource utilization seems particularly important. A typical example of such scenarios is the deployment of few relays in ad hoc networks or the transmissions between the hot-spot nodes located at the edge of the cell [3].

Recently, auction-theoretic approaches to resource allocation problems have been explored in wireless cooperative networks [4], [5]. The auction mechanism is beneficial to allocating available resources among multiple bidders in a distributed way, with uncertainty about the individual information, and achieving competitive fairness. In this work, we propose a relay power allocation scheme for multiple node-pairs relay wireless networks by using auction mechanism. The main contributions are three-fold:

1) We derive a refined energy efficiency metric, which is characterized by easy implementation and adequately shows the relationship between the QoS provisioning and the energy consumption.

2) We design an auction mechanism for relay power allocation by modeling the source-destination pairs as bidders, the relay as an auctioneer, and the available energy of relay as a traded resource. Then, we treat the proposed auction mechanism as an auction game, and prove the existence, uniqueness and Pareto optimality of the Nash equilibrium (NE). According to the proposed energy efficiency metric, the resulting NE can yield the energy-efficient relay power allocation strategies.

3) We propose a distributed algorithm based on best-response functions to reach the Pareto optimal NE of our auction game. Moreover, we analyze the convergence and provide the upper bound of the convergence. Interestingly, the convergence is only dependent on the number of source-destination pairs and independent of the initial bidding strategies and the available energy of the relay.

II. Network Model and Problem Formulation

A. Network Model

We consider a multiple node-pairs relay wireless network, where the source node s_i communicates with its corresponding destination node d_i, $i = 1, \ldots, N$, and one relay r is available to assist the data transmissions from N source-destination pairs. Orthogonal transmissions are assumed among different nodes for simultaneous communications of all source-destination pairs.

Taking the node-pair i for example, the cooperative communication scheme is carried out by two phases, namely a local transmission and a long-haul cooperative transmission. For both phases, the channels are modeled by a path loss exponent δ and frequency flat Rayleigh fading. More precisely, $h_{s_i,r} \sim \mathcal{CN}(0,1)$, $h_{s_i,d_i} \sim \mathcal{CN}(0,1)$, and $h_{r,d_i} \sim \mathcal{CN}(0,1)$ are unitary power, Rayleigh fading channel coefficients. Without loss of generality, each node has the same Gaussian noise variance σ^2. Here, we adopt the outage performance as a QoS measure. In order to guarantee the basic QoS requirement, the outage probability $P_{out,i}$ should not be larger than the threshold value $P_{out,i}^{thr}$, under the fixed outage capacity $\mathcal{C}_{out,i}$.

During Phase 1, there exists the channel reciprocity between s_i and r, and the relay r is expected to cooperate during Phase 2. Hence, setting the rate of Phase 2 to $\mathcal{C}_{out,i}$, the capacity region of Phase 1 is constrained to:

$$\frac{1}{2}\log_2\left(1+\frac{p_{s_i,r}}{\sigma^2}\left(l_i^{\text{local}}\right)^{-\delta}|h_{s_i,r}|^2\right) \geq \mathcal{C}_{out,i}, \quad (1)$$

where $p_{s_i,r}$ is the transmit power, and l_i^{local} represents the local distance between the source node s_i and the relay r, respectively. Moreover, we normalize a unit of time scale to 1, and derive the energy consumption for Phase 1 as $E_i^1 = (p_{s_i,r}+p_{ct}+p_{cr})/2$, where p_{ct} and p_{cr} denote the transmitter and the receiver circuit power, respectively. For simplicity, all nodes have the same p_{ct} and p_{cr}. Specifically, the part of s_i is $(p_{s_i,r}+p_{ct})/2$, and the part of r is $p_{cr}/2$.

During Phase 2, there is no channel reciprocity, and thus, s_i and r jointly transmit data to d_i based on the Alamouti codes. Then, each transmitting member has the same transmit power [6], namely $p_{s_i,d_i} = p_{r,d_i} = p_i^{\text{co}}$. Hence, the energy consumption for Phase 2 is formulated as $E_i^2 = p_i^{\text{co}} + p_{ct}$, where the part of s_i is $(p_{s_i,d_i}+p_{ct})/2$, and the part of r is $(p_{r,d_i}+p_{ct})/2$. From [6], the outage probability is

$$P_{out,i} = \Gamma\left(2, \frac{\left(2^{2\mathcal{C}_{out,i}}-1\right)\sigma^2 l_i^\delta}{p_i^{\text{co}}}\right), \quad (2)$$

where $\Gamma(2,m) = \int_0^m xe^{-x}dx$, and l_i is the distance of the i^{th} long-haul cooperative transmission link.

B. Design of Energy Efficiency Metric

As we know, the bit-per-energy efficiency is a widely-used metric in terms of the energy efficiency design [2], and well captures the tradeoff between the throughput and the energy consumption. However, throughput is not an appropriate measure for all the communication networks, in particular for multimedia communications [7]. This motives us to propose a refined energy efficiency metric to precisely capture the characteristics of the application-driven transmission system. In this regard, we formulate the ratio of QoS provisioning to the energy costs as our energy efficiency metric.

As outage performance is related to large scale channel fading and statistical small scale fading, it is appropriate for the model description which lacks instantaneous channel knowledge. Therefore, it is adopted as a QoS representative in this work. To this end, for the node-pair i, we have

$$\text{Type I}: \qquad \frac{\alpha_i\left(P_{out,i}^{thr}-P_{out,i}\right)}{E_i^1+E_i^2}, \quad (3)$$

where α_i is a positive constant. However, the use of Type I may increase the computational complexity, which motivates us to rewrite (3) as

$$\text{Type II}: \qquad \beta_i\left(P_{out,i}^{thr}-P_{out,i}\right)-\left(E_i^1+E_i^2\right), \quad (4)$$

where β_i is also a positive constant. In particular, both α_i and β_i are tunable parameters, which is used to adjust the steepness, and achieve the fairness and heterogeneous service requirements.

Proposition 1: There exists $\beta_i = \varrho \cdot \alpha_i$ (where $\varrho > 0$ is a scaling factor) enabling Type I and Type II to achieve their optimal values at the same allocated power. That is, Type I and Type II are equivalent in this case.

Indeed, Proposition 1 can be proved by first setting the derivatives of (3) and (4) with respect to p_i^{co} as zero, then finding the relationship between α_i and β_i when (3) and (4) achieve the optimal solutions at the identical p_i^{co}. Hence, (4) can be treated as an energy efficiency metric. Particularly, Type II can simplify the derivation analysis, and facilitate resource allocation strategy updates. Moreover, in order to consume energy as little as possible, $p_{s_i,r}$ can be obtained by transforming (1) into the equation, i.e., $p_{s_i,r} = \left(2^{2\mathcal{C}_{out,i}}-1\right)\left(l_i^{\text{local}}\right)^\delta \sigma^2 \Big/ |h_{s_i,r}|^2$. *For Type II, achieving the energy efficiency of the node-pair i is equivalent to maximizing (4) in terms of p_i^{co}.*

III. Auction-based Relay Power Allocation for Cooperative Networks

A. Auction Mechanism Design

In our model, each source-destination pair competes for relaying assistance, i.e., relay transmit power, and is only interested in its own action and its resulting benefit. Hence, we resort to the notion of competitive fairness [8] to achieve the fair allocation design for competitive scenarios. As a notable example of competitive fairness, the auction approach is exploited to determine relay power allocation in this work. Specifically, in the auction mechanism for relay power allocation, the source-destination pairs constitute a set of risk-neutral bidders, denoted by $\mathcal{N} = \{1,\ldots,N\}$, and the relay r acts as the auctioneer. The object for sale is the available energy of the relay r. Moreover, the private value of the bidder i, V_i, which represents the value of the sale object to itself, should exhibit two measures of interest, namely, the amount of cooperation benefits and costs. Here, the two measures for bidder i are characterized by its outage performance improvement and the sum of energy consumption for Phase 1 and Phase 2, respectively. To this end, the private value of bidder i can be expressed as

$$V_i = \eta_i\left(P_{out,i}^{thr}-P_{out,i}\right)-\left(\frac{1}{2}p_{s_i,r}+\frac{1}{2}p_{s_i,d_i}+p_{ct}\right), \quad (5)$$

where η_i is a positive constant. The bidder i submits its bid b_i to r, and then, obtains a portion of the relay transmit power

$$p_{r,d_i} = \frac{b_i}{\sum_{j\in\mathcal{N}} b_j + \theta} p_r, \tag{6}$$

where p_r is the available transmit power of the relay r, and θ is the reserved bid which is a positive bid placed by the relay r. The relay r adjusts its reserved bid according to the variation of energy efficiency demand. Moreover, the bidder i pays for the help of the relay r. The payment should be proportional to the amount of energy that r dedicates to serving the bidder i, that is,

$$C_i = \frac{1}{2}\mu\left(p_{r,d_i} + p_{cr} + p_{ct}\right), \tag{7}$$

where μ is the unit price which the relay r announces to all bidders at the beginning of the auction.

B. Nash Equilibrium Under Auction Game

The essence of an auction is a game, where the players are the bidders, the strategies are the bids, and both allocations and payments are functions of the bids. Accordingly, the proposed auction mechanism can be compactly represented in strategic form as $G = \left\langle \mathcal{N}, \{\mathcal{B}_i\}_{i\in\mathcal{N}}, \{u_i\}_{i\in\mathcal{N}} \right\rangle$, which specifies for each bidder i a set of bidding strategies $\mathcal{B}_i$ ($b_i \in \mathcal{B}_i$), and a utility function $u_i(b_1, \ldots, b_N)$ associated with outcome of the auction arising from a bidding strategy profile $\mathbf{b} = (b_1, \ldots, b_N)$. Moreover, the joint strategy space is $\mathcal{B} = \mathcal{B}_1 \times \ldots \times \mathcal{B}_N$. For $\forall \mathbf{b} \in \mathcal{B}$, we can rewrite $\mathbf{b} = (b_i, \mathbf{b}_{-i})$, where $\mathbf{b}_{-i} = (b_1, \ldots, b_{i-1}, b_{i+1} \ldots, b_N)$ denotes the bidding strategies for all $N-1$ others excluding bidder i. Hence, the utility for the bidding strategy profile $\mathbf{b}$ is

$$u_i(b_i, \mathbf{b}_{-i}) = V_i(b_i, \mathbf{b}_{-i}) - C_i(b_i, \mathbf{b}_{-i}), \tag{8}$$

Formally, our auction game G is defined as

$$\max_{b_i\in\mathcal{B}_i} \quad u_i(b_i, \mathbf{b}_{-i}), \quad \text{for all } i \in \mathcal{N}. \tag{9}$$

Our work mainly concentrates on the relay power allocation, hence, we take the single-relay case for example. The proposed auction-based relay power allocation scheme can be extended to networks with multiple relays. The key difference between them is to add the step of each node-pair choosing its appropriate relay. Consequently, if there exist N_r relays, the multi-relay network can be divided into $N_r + 1$ clusters of nodes, and each cluster is independently analyzed as a single-relay case. The readers can be referred to [5] for more details.

Proposition 2: The outcome of the proposed auction game G can result in the energy-efficient relay power allocation.

Proof: Note that the utility function in (8) is almost identical to the energy efficiency metric in Type II, and the additional parameters, η_i and μ, do not affect the relationship between the energy efficiency and relay transmit power (The reason is similar to Proposition 1.). Moreover, each bidder i of our auction game G is to maximize the utility function in terms of b_i. p_i^{co} is a monotonically increasing function with b_i when $\mathbf{b}_{-i}$ is fixed, and thus, the solution which maximizes the utility function in terms of p_i^{co} coincides with the outcome of G. Since achieving the energy efficiency in Type II is equivalent to maximizing (4) in terms of p_i^{co}, the proof is complete. ■

Considering that the NE is treated as a desirable outcome of any multi-player game, we are interested in the relay power allocation which results from the NE strategies $\mathbf{b}^*$.

Theorem 1: A NE exists for the proposed auction game G.

Proof: We rewrite (8) as $u_i(b_i, \mathbf{b}_{-i}) = Y_i(b_i, \mathbf{b}_{-i}) - X_i(b_i, \mathbf{b}_{-i})$, where $Y_i(b_i, \mathbf{b}_{-i}) = \eta_i \left(P_{out,i}^{thr} - \Gamma\left(2, \frac{\left(2^{2\mathcal{C}_{out,i}}-1\right)\sigma^2 l_i^\delta}{p_i^{\text{co}}(b_i, \mathbf{b}_{-i})}\right)\right)$, and $X_i(b_i, \mathbf{b}_{-i}) = \frac{1}{2}\left[(1+\mu)\, p_i^{\text{co}}(b_i, \mathbf{b}_{-i}) + p_{s_i,r} + \mu p_{cr} + (2+\mu)\, p_{ct}\right]$. By taking the second derivative of $Y_i(b_i, \mathbf{b}_{-i})$ with respect to b_i, we find that if $p_i^{\text{co}}(b_i, \mathbf{b}_{-i}) \geq \left(2^{2\mathcal{C}_{out,i}}-1\right)\sigma^2 l_i^\delta$, $\partial^2 Y_i(b_i, \mathbf{b}_{-i}) / \partial b_i^2 \leq 0$. Noting that $\left(2^{2\mathcal{C}_{out,i}} - 1\right)\sigma^2 l_i^\delta$ is tiny (e.g., it asymptotically achieves to zero), the above constraint on $p_i^{\text{co}}(b_i, \mathbf{b}_{-i})$ can be satisfied. Then, $Y_i(b_i, \mathbf{b}_{-i})$ is concave. Also, $X_i(b_i, \mathbf{b}_{-i})$ is convex. As a result, the utility function u_i is concave, which in turn implies quasi-concavity. Moreover, $\mathcal{B}_i$ is a non-empty, compact and convex set. Hence, the conditions of the existence of NE, as in [9], are satisfied, and accordingly G has a NE at least. ■

Next, we discuss the properties of NE. First, we derive the derivative of $u_i(b_i, \mathbf{b}_{-i})$ with respect to b_i by

$$\frac{\partial u_i(b_i, \mathbf{b}_{-i})}{\partial b_i} = \left[\frac{\eta_i m_i^3 e^{-m_i}}{M_{1,i}} - \frac{1}{2}(1+\mu)\right] \frac{\left(\sum_{j\neq i} b_j + \theta\right) p_r}{\left(\sum_{j\in\mathcal{N}} b_j + \theta\right)^2}, \tag{10}$$

where $M_{1,i} = \left(2^{2\mathcal{C}_{out,i}} - 1\right)\sigma^2 l_i^\delta$, and $m_i = \frac{\left(2^{2\mathcal{C}_{out,i}}-1\right)\sigma^2 l_i^\delta}{p_i^{\text{co}}(b_i, \mathbf{b}_{-i})}$. Notice that the second term on the right-hand side of (10) is strictly positive. In order to find the best-response strategies, we have $\left((M_{2,i})^{-1/3} m_i\right)^3 = e^{m_i}$, where $M_{2,i} = \frac{(1+\mu)M_{1,i}}{2\eta_i}$. By using the principal branch of the Lambert W function which is denoted by $W_0(\cdot)$, we obtain

$$p_i^{\text{co}} = \frac{M_{1,i}}{-3W_0\left(-\frac{1}{3}(M_{2,i})^{1/3}\right)}. \tag{11}$$

Hence, the best-response function can be expressed as

$$BR_i(b_i, \mathbf{b}_{-i}) = \frac{p_i^{\text{co}}\left(\sum_{j\neq i} b_j + \theta\right)}{p_r - p_i^{\text{co}}}. \tag{12}$$

Then, we can define the best-response vector function $BR(\mathbf{b})$ as $BR(\mathbf{b}) = (BR_1(\mathbf{b}), \ldots, BR_N(\mathbf{b}))$, and its components are given by (12).

Definition 1 [10]: If a best-response vector function $BR(\cdot)$ satisfies the following condition: for all $\psi > 1$, $(1/\psi)\,\mathbf{b} \leq \tilde{\mathbf{b}} \leq \psi\mathbf{b}$ implies $(1/\psi)\, BR(\mathbf{b}) < BR\left(\tilde{\mathbf{b}}\right) < \psi BR(\mathbf{b})$, it is said to be two-sided scalable.

Theorem 2 [10]: If a best-response vector function $BR(\cdot)$ is two-sided scalable and a fixed point exists, then that fixed point is unique.

Theorem 3: The proposed auction game G has a unique NE.

Proof: According to Theorem 1, our auction game G has a NE $\mathbf{b}^*$, and then, $\mathbf{b}^*$ is a fixed point of the best-response vector function, i.e., $\mathbf{b}^* = BR(\mathbf{b}^*)$. For $\forall i \in \mathcal{N}$ and all $\psi > 1$, if $(1/\psi)\,\mathbf{b} \leq \tilde{\mathbf{b}} \leq \psi\mathbf{b}$, by monotonicity, we can get $BR_i\left((1/\psi)\,\mathbf{b}\right) \le BR_i\left(\tilde{\mathbf{b}}\right) \le BR_i\left(\psi\mathbf{b}\right)$. From (12), we obtain $\psi BR_i\left((1/\psi)\,\mathbf{b}\right) > BR_i\left(\psi\left((1/\psi)\,\mathbf{b}\right)\right) = BR_i\left(\mathbf{b}\right)$. These above inequalities jointly result in $(1/\psi)\,BR_i\left(\mathbf{b}\right) < BR_i\left(\tilde{\mathbf{b}}\right) < \psi BR_i\left(\mathbf{b}\right)$. Accordingly, $BR\left(\cdot\right)$ is two-sided scalable. By using Theorem 2, we conclude that the NE $\mathbf{b}^*$ is unique. ■

Theorem 4: The relay power allocation solution deriving from the unique NE of the proposed auction game G is Pareto optimal.

Proof: For the proposed auction game G, we assume that the bidding strategy profile $\mathbf{b}$ is the outcome of the utilitarian social welfare problem which is defined as

$$\max_{\mathbf{b}} \quad \sum_{i=1}^{N} u_i\left(\mathbf{b}\right), \tag{13}$$

but not Pareto optimal. Then, there exists a certain strategy profile $\tilde{\mathbf{b}}$ such that $u_i\left(\tilde{\mathbf{b}}\right) \ge u_i\left(\mathbf{b}\right)$, for all $i \in \mathcal{N}$ and $u_i\left(\tilde{\mathbf{b}}\right) > u_i\left(\mathbf{b}\right)$ for some $i \in \mathcal{N}$. This implies that $\sum_{i=1}^{N} u_i\left(\tilde{\mathbf{b}}\right) > \sum_{i=1}^{N} u_i\left(\mathbf{b}\right)$. As a result, $\mathbf{b}$ cannot be a solution to the problem in (13), which contradicts the original assumption. Therefore, it must be a Pareto optimal point.

Furthermore, the solution of (13) should satisfy the following $|\mathcal{N}|$ equations: $\partial\left(\sum_{i=1}^{N} u_i\left(\mathbf{b}\right)\right)\big/\partial b_i = 0$, for all $i \in \mathcal{N}$. The result of NE satisfies this case (Please refer to the analysis of best-response function for the details.). Therefore, the resulting NE $\mathbf{b}^*$ is a solution to the problem in (13), that is, it is Pareto optimal. ■

C. Distributed Algorithm for Auction Game

In this subsection, we propose a relay power allocation algorithm to reach the NE of G in a distributed way, and the details are shown in Table I. Specifically, the bidders iteratively update their bidding strategies based on best-response functions, and the iteration vector function can be expressed as

$$\mathbf{b}^{(n+1)} = BR\left(\mathbf{b}^{(n)}\right), \tag{14}$$

whose component is given by $b_i^{(n+1)} = \frac{p_i^{\mathrm{co}}\left(\sum_{j\neq i} b_j^{(n)}+\theta\right)}{p_r - p_i^{\mathrm{co}}}$, for $\forall i \in \mathcal{N}$, where p_i^{co} is given by (11). In particular, the proposed algorithm favors distributing the available energy of the relay, with uncertainty about the individual information (e.g., the utility value, the chosen strategies, etc.).

In fact, [10, Theorem 10] shows that if the best-response function is two-sided scalable and a fixed point exists, this fixed point can be reached via using its corresponding iteration vector function. Hence, we conclude that the proposed algorithm can converge to the NE $\mathbf{b}^*$. Moreover, we investigate the convergence time of the proposed algorithm, which is necessary and important for communication quality.

Note that if $\mathbf{b}^{(n)}$ is a NE, then $\mathbf{b}^{(n+1)} = \mathbf{b}^{(n)}$ and the allocated power stops changing. In order to characterize the difference between each iteration during the auction process, we define the potential function as $\Phi\left(\mathbf{b}\right) = \sum_{i=1}^{N}\left(b_i - \overline{b}\right)^2$, where $\overline{b} = \frac{1}{N}\sum_{i=1}^{N} b_i$.

Table I: Distributed Relay Power Allocation Algorithm

begin
initialization
$n = 1$;
r announces the information, i.e., the reserved bid θ, the price μ, and its total transmit power p_r to all bidders;
$\forall i$ determines the transmit power $p_{s_i,r}$ by transforming (1) into the equation, chooses a feasible bid $b_i^{(1)}$ arbitrarily, and submits $b_i^{(1)}$ to r;
while $(n > 0)$
r broadcasts the sum of bids of all bidders $\sum_{i\in\mathcal{N}} b_i^{(n)}$;
while $(\forall i \in \mathcal{N})$
i calculates its updated bidding strategy $b_i^{(n+1)}$ according to (14), and submits it to r;
end while
if $(\mathbf{b}^{(n+1)} = \mathbf{b}^{(n)})$
$n = -1$;
else
$n = n + 1$;
end if
end while

Lemma 1: Let $\mathbf{b}^{(n)} = \left(b_1^{(n)}, ..., b_N^{(n)}\right)$ be the bidding strategy profile at iteration n by using the proposed algorithm, and we have $\mathbb{E}\left[\Phi\left(\mathbf{b}^{(n+1)}\right)|\mathbf{b}^{(n)}\right] \le N + 2\sqrt{N\cdot\Phi\left(\mathbf{b}^{(n)}\right)}$.

Proof: Let $z\left(\mathbf{b}\right) = \sum_{i=1}^{N}\sum_{i'=1}^{N}|b_i - b_{i'}|$, and $d_i\left(\mathbf{b}\right) = \frac{1}{N}\sum_{i'=1}^{N}\left(b_i - b_{i'}\right)$. By using the upper-bound of $z\left(\mathbf{b}\right)$ and $d_i\left(\mathbf{b}\right) \le 1$, we have

$$\mathbb{E}\left[\Phi\left(\mathbf{b}^{(n+1)}\right)|\mathbf{b}^{(n)}\right] \le N + \frac{1}{N}\sum_{i=1}^{N}\sum_{i'=1}^{N}\left|b_i^{(n)} - b_{i'}^{(n)}\right|. \tag{15}$$

Moreover, the right-hand side of (15) is $N + 2\sum_{i=1}^{N}\left|b_i^{(n)} - \overline{b}\right|$ at most. By Cauchy-Schwarz, we get $\mathbb{E}\left[\Phi\left(\mathbf{b}^{(n+1)}\right)|\mathbf{b}^{(n)}\right] \le N + 2\sqrt{N\cdot\Phi\left(\mathbf{b}^{(n)}\right)}$. ■

Lemma 2: Suppose that the bidding strategy satisfies $\left|b_i - \overline{b}\right| \ge \varepsilon$ $(\varepsilon > 0)$ and $\left|b_i - \overline{b}\right| \le \left|b_N - \overline{b}\right|$, for all $i \in \mathcal{N}$. Let $\mathbf{w} = \left(w_1, ..., w_{N-1}\right)$ with $w_i = b_i$ for $i \in [1, N-1]$. Then, $\Phi\left(\mathbf{b}\right) - \frac{1}{2N}z\left(\mathbf{b}\right) \ge \Phi\left(\mathbf{w}\right) - \frac{1}{2(N-1)}z\left(\mathbf{w}\right)$.

Proof: Let $f\left(g\right) = \sum_{i=1}^{N-1}\left(b_i - g\right)^2$. Note that the derivation of $f\left(g\right)$ is:

$$\frac{\partial f\left(g\right)}{\partial g} = 2\left(N-1\right)g - 2\left(N-1\right)\overline{w}, \tag{16}$$

where $\overline{w}$ is the average value of all the elements of $\mathbf{w}$. Moreover, the second derivative is $\partial^2 f\left(g\right)\big/\partial g^2 = 2\left(N-1\right) \ge 0$. Hence, $f\left(g\right)$ is minimized at $g = \overline{w}$, and then, we achieve

$$\Phi\left(\mathbf{b}\right) - \Phi\left(\mathbf{w}\right) = k^2 + \sum_{i=1}^{N-1}\left(b_i - \overline{b}\right)^2 - \sum_{i=1}^{N-1}\left(b_i - \overline{w}\right)^2 \ge k^2. \tag{17}$$

Next, we assume that $b_N = \overline{b} + k$ and $S\left(\mathbf{b}\right) = \sum_{i=1}^{N}\left|b_i - \overline{b}\right|$, then $S\left(\mathbf{b}\right) - S\left(\mathbf{w}\right) \le 2Nk$. Let $M = \lceil b_N - 2k\rceil$ and $y_j = \left|\{i|\, b_i = j\}\right|$. Thus, we have $b_i \ge b_N - 2k$, $y_j = 0$ for $j > b_N$, and $y_j = 0$ for $j < M$. Similarly, let $B\left(\mathbf{b}\right) = \sum_{j=M}^{b_N} y_j\left(y_{j-1} - y_{j+1}\right)^2$, and we can further get

$$B\left(\mathbf{b}\right) - B\left(\mathbf{w}\right) \le 2y_{b_N-k}\left(N - y_{b_N-k}/2\right). \tag{18}$$

Since the right-hand side of (18) is at most N^2, we can obtain:

$$\frac{1}{2N}z(\mathbf{b})-\frac{1}{2(N-1)}z(\mathbf{w}) \leq \frac{S(\mathbf{b})-S(\mathbf{w})}{N}-\frac{B(\mathbf{b})-B(\mathbf{w})}{N^2} \leq 2k-1. \quad (19)$$

In the case of $b_N = \bar{b} - k$, the proof is similar to the above. Therefore, the proof is complete. ∎

Theorem 5: Let N be the number of the source-destination pairs in the wireless cooperative network, and T be the number of rounds taken by the proposed algorithm to reach a NE for the first time. Then, $\mathbb{E}[T] \sim O(N^2+N)$.

Proof: Note that the relationship between $\mathbf{b}^{(n+1)}$ and $\mathbf{b}^{(n)}$ satisfies: $\mathbb{E}\left[\Phi\left(\mathbf{b}^{(n+1)}\right)\right] \leq N + 2\sqrt{N\cdot\mathbb{E}\left[\Phi(\mathbf{b}^{(n)})\right]}$. Then, let $\tau = O(N)$, $\Upsilon^{(n)} = \Phi\left(\mathbf{b}^{(n+\tau)}\right) - \sum_i d_i\left(\mathbf{b}^{(n)}\right)$, and $\Omega^{(n)} = \min\left(\Upsilon^{(n)}, z\left(\mathbf{b}^{(n)}\right)\right)$. From Lemma 2, we know that $\Upsilon^{(n)}$ is a super-martingale. Hence,

$$\mathbb{E}\left[\Omega^{(n+1)}\left|\Omega^{(n)} < z\left(\mathbf{b}^{(n)}\right)\right.\right] \leq \Upsilon^{(n)} = \Omega^{(n)}, \quad (20)$$

and

$$\mathbb{E}\left[\Omega^{(n+1)}\left|\Omega^{(n)} = z\left(\mathbf{b}^{(n)}\right)\right.\right] \leq z\left(\mathbf{b}^{(n)}\right) = \Omega^{(n)}. \quad (21)$$

If $\mathbf{b}^{(n)}$ is not a NE, $P\left[\Phi\left(\mathbf{b}^{(n+1)}\right)\neq\Phi\left(\mathbf{b}^{(n)}\right)\left|\Phi\left(\mathbf{b}^{(n)}\right)\right.\right]\geq \left(\Phi\left(\mathbf{w}^{(n)}\right)-\frac{z\left(\mathbf{w}^{(n)}\right)}{2(N-1)}\right)\Big/\Phi\left(\mathbf{w}^{(n)}\right)$. Based on Lemma 2, we have i) for $x > 0$, $P\left(\Upsilon^{(n+1)} \neq \Upsilon^{(n)}\left|\Upsilon^{(n)} = x\right.\right) \geq \bar{b}\Big/\max_{i\in\mathcal{N}} b_i$, ii) for $0 < x < z\left(\mathbf{b}^{(n)}\right)$, $P\left(\Omega^{(n+1)} \neq \Omega^{(n)}\left|\Omega^{(n)} = x\right.\right) \geq \bar{b}\Big/\max_{i\in\mathcal{N}} b_i$. Consequently, we can show

$$\mathbb{E}\left[\left(\Omega^{(n+1)} - \Omega^{(n)}\right)^2\left|0 < \Omega^{(n)} < z\left(\mathbf{b}^{(n)}\right)\right.\right] \geq \bar{b}. \quad (22)$$

In fact, T can be viewed a stopping time. Define

$$\Xi^{(n)} = \left(z\left(\mathbf{b}^{(n)}\right) - \Omega^{(n)}\right)^2 - \bar{b}\cdot T, \quad (23)$$

and we find that $\Xi^{(n\wedge T)}$ is a sub-martingale. Let P_1 be the probability for $\Omega^{(n)} = 0$, that is, $\mathbf{b}^{(n+\tau)}$ is a NE. Using Optional Stopping Theorem [11], we get

$$P_1 z\left(\mathbf{b}^{(n)}\right)^2 - \bar{b}\mathbb{E}[T] = \mathbb{E}\left[\Xi^{(T)}\right] \geq \Xi^{(1)} > 0. \quad (24)$$

By means of the Estimation Theorem [11], we have $\mathbb{E}[T] \sim O(N) + P_1 z\left(\mathbf{b}^{(n)}\right)^2\Big/\bar{b}$. For $\Omega^{(n)} = 0$, according to Lemma 1, we have $\mathbb{E}\left[T\left|\Omega^{(T)} = 0\right.\right] \sim O(N) + z\left(\mathbf{b}^{(n)}\right)^2\Big/\bar{b} \sim O\left(N+N^2\right)$. Similarly, for $\Omega^{(n)} = z\left(\mathbf{b}^{(n)}\right)$, we also have $\mathbb{E}\left[T\left|\Omega^{(T)} = z\left(\mathbf{b}^{(n)}\right)\right.\right] \sim O\left(N+N^2\right)$. Therefore, we can get $\mathbb{E}[T] \sim O\left(N+N^2\right)$. ∎

IV. Numerical Results

The simulation setup is as follow: several source-destination nodes are randomly located around the relay. For both the local broadcasting and the long-haul cooperative channels, Rayleigh fading coefficients are modeled as unitary power, complex Gaussian random variables. The constant κ is set to 1, and

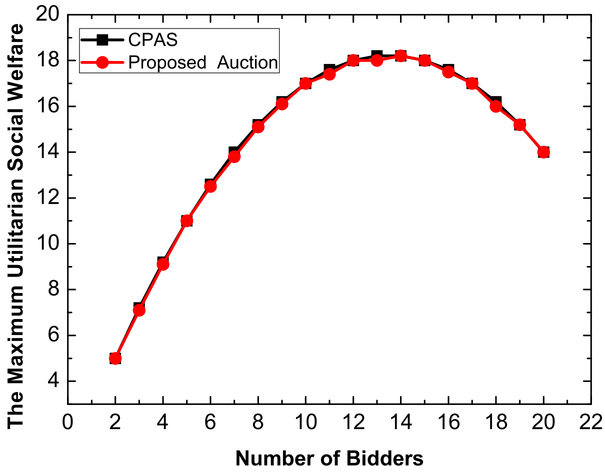

Fig. 1. Performance comparison in terms of different number of bidders (The available transmit power of the relay is 2W.).

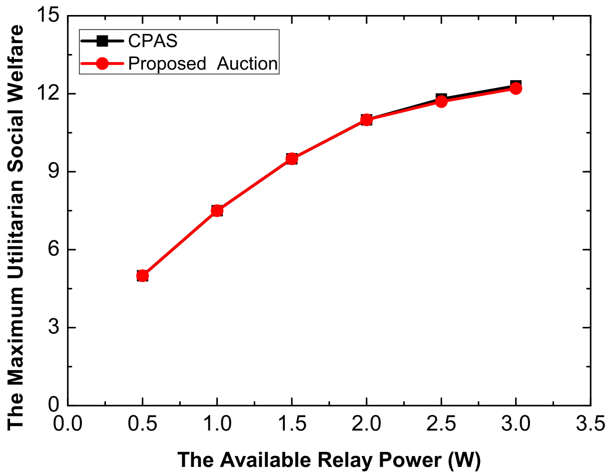

Fig. 2. Performance comparison in terms of different available relay power values (The number of the bidders is 5.).

the path loss exponent δ is set to 3. The Gaussian noise variance σ^2 is 10^{-12}W. For simplicity, we take the outage capacity $\mathcal{C}_{out,i}$ and the threshold value of outage probability $P_{out,i}^{thr}$ as the same values for all source-destination nodes, i.e., $\mathcal{C}_{out,i} = 1.4$bps/Hz and $P_{out,i}^{thr} = 10^{-4}$ [6]. Besides, we choose $\theta = 1$ for all the simulations as in [5].

For illustrative purpose, we compare our scheme with the centralized power allocation scheme (CPAS) which achieves the optimal solution [11]. Fig. 1 depicts the maximum utilitarian social welfare with different number of the bidders, and Fig. 2 shows the case of different available relay power values. Note that these results are obtained using 50 runs in order to obtain statistically meaningful average values. From the given figures, we can find that our scheme almost achieves the same performance as that of CPAS, irrespective of the number of bidders and available relay power values. In other words,

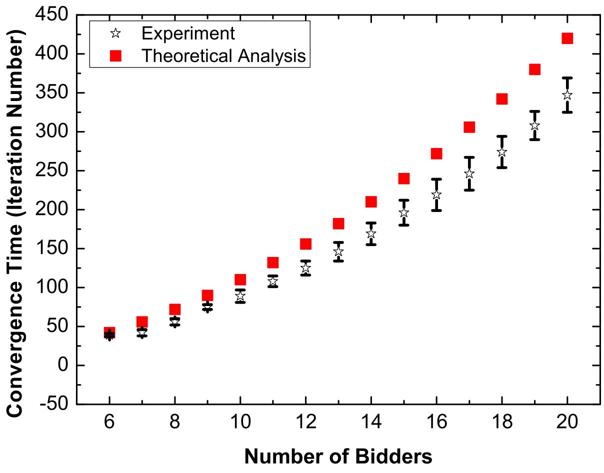

Fig. 3. The convergence time as the number of bidders varies from 6 to 20.

the resulting NE is a solution of maximizing the utilitarian social welfare. Hence, the resulting NE via using the proposed distributed algorithm is Pareto optimal.

Moreover, Fig. 3 shows the convergence time as the number of bidders varies from 6 to 20 with random relay transmit power. The more the bidders, the slower the convergence speed. That is because the increase in the number of bidders may result in the growth of opportunity of interaction among the bidders. Also, we verify the convergence performance when the available transmit power of the relay is a dynamic value. We find that the convergence time is strongly dependent on the number of the bidders, and independent of the initial bidding strategies and the available transmit power of the relay node. Most notably, when N is small, Theorem 5 provides an accurate estimation, while as N goes large, Theorem 5 tends to become a bit of lose. For example, when $N = 6$, the theoretical upper bound is 42, and the practical convergence round is 39. These numbers change to 420 and 347 when $N = 20$. This is due to the fact that we use a conservative estimation of $\Phi(\mathbf{b})$ in Lemma 2, and we refer the readers to [11, Theorems 3.1-3.4] for more details on the performance deviation by using this estimation method.

V. Conclusions

In this work, we develop an auction-based relay power allocation scheme for multiple node-pairs relay wireless networks. Firstly, we design a refined energy efficiency metric to describe the relationship between the QoS provisioning and the energy consumption. Then, we propose an auction mechanism for relay power allocation. From the game theoretic perspective, we prove the existence, uniqueness, and Pareto optimality of the NE for our auction game. From the resulting NE, we can determine the energy-efficient allocation strategies. Moreover, we develop a distributed algorithm to converge to the Pareto optimal NE with the incomplete individual information. Interestingly, we find that the convergence is only dependent on the number of source-destination pairs.

Acknowledgement

This work is supported by the NSF of China (Grant No. 60972051, 61001107), and the Major National Science & Technology Specific Projects (Grant No. 2010ZX03006-002-04).

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