On Energy Equilibrium in Wireless Sensor Networks

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***Abstract*—In this paper, we formulate the energy equilibrium problem as an optimal corona division, where data fusion is considered in data gathering. For a circular multi-hop sensor network with uniform node distribution, we demonstrate that the maximum energy equilibrium for a given circular area can be achieved only if the area increases in geometric progression from the outer corona to the neighboring inner corona except for the outermost one. The approach for computing the optimal parameters is presented in terms of maximizing the network lifetime. Based on the mathematical model, we propose an energy equilibrium routing(EER). Our simulation results demonstrate that EER can effectively achieve energy equilibrium.**

***Index Terms*—Wireless Sensor Network; Energy Equilibrium; Corona Division; Data Fusion**

I. INTRODUCTION

Most sensor nodes are unable to communicate with the sink node directly because of their limited communication capacity. As a result of this limitation, multi-hop routing has become the basic method for data gathering in Wireless Sensor Networks (WSNs) [1], [2], [3], [4]. In this way, intermediate nodes deplete their energy faster when they take more tasks, such as relaying the received data or completing the fusion process to curtail the network load [5]. Therefore, the energy hole that can occur prevents remote data from being sent further to the sink node [6]. The avoidance of the energy hole in WSNs has attracted a lot of attention and some valuable results have been reported in recent years. Powell et al. proposed a centralized algorithm to compute the optimal parameters for WSNs and proved that these parameters maximize the network lifetime [7]. Olariu et al. investigated theoretical aspects of the uneven energy depletion phenomenon in sink-based wireless sensor networks [8]. Wu et al. explored the theoretical aspects of the non-uniform node distribution strategy that addresses the energy hole problem in WSNs. They proposed a distributed shortest path routing algorithm tailored for the proposed non-uniform node distribution strategy [9]. Zhang et al. formulate the energy consumption balancing problem as an optimal transmitting data distribution problem by combining the ideas of corona-based network division and mixed-routing strategy together with data aggregation [10].

In this work, we study how to achieve maximal energy equilibrium for a given area in WSNs with uniform node distribution. Similar to reference [10], we divide the problem into two sub-problems: intra-corona equilibrium and inter-coronas equilibrium. The remainder of this paper is organized as follows: Section 2 presents the system models and the statement of the problem. Section 3 gives the solution for the energy equilibrium of intra-corona and inter-coronas. Section 4 describes the design of EER. In section 5, energy optimization and the cost for energy equilibrium are studied. In Section 6, EER is verified with extensive simulations. Finally, we make some concluding remarks in Section 7.

II. SYSTEM MODELS AND PROBLEM STATEMENT

A. System Models

We assume that all the nodes are uniformly distributed in the circular area A of radius R with node distribution density ρ . Only one sink node is located at the center of the area A. All the nodes have the same initial energy. In data gathering, the maximum communication distance is also the same for all nodes. The data gathering process is divided into rounds. In each round, the sensor nodes complete data reception, fusion, and then transmit the data to the sink node using multi-hop routing. All the nodes generate data at the same rate, which is L bit/s.

Similar to reference [11], a data fusion model is employed in this study, where the node u in corona C_i needs to receive the data sent from node v in corona C_{i+1}, marked as $D(v)$. The total data after data reception and fusion with its own data is expressed as:

$$\tilde{D}(u) = max(D(u), D(v)) + min(D(u), D(v))(1 - \sigma) \quad (1)$$

where $D(u)$ and $\tilde{D}(u)$ represents the data amount before and after data fusion. σ represents the correlation coefficient of data. Similar to [12], the energy spent by transmitting 1 bit of data over distance d is $e_t(d) = \varepsilon_{elec} + \varepsilon_{amp} d^k$, where ε_{elec} and ε_{amp} are both system parameters. The energy dissipation when receiving data is $e_t(d) = \varepsilon_{elec}$. Data fusion can introduce an extra energy consumption, which is denoted by e_f.

B. Problem Statement

The concentric coronas, named as C_1, C_2, ...,C_N are obtained by dividing the circle A using the sink node as the center node. The area of C_i is denoted by $S_i(1 \leq i \leq N)$. Definition 1. The Equilibrium ratio, denoted by P_e, is defined as the ratio of the area where we can achieve energy equilibrium to the total area.

$$P_e = \frac{S_e}{S_e + S_u} \tag{2}$$

S_u and S_e represents the area where we cannot realize energy equilibrium and the area with the energy equilibrium, respectively. Our optimization objective is to achieve maximal energy equilibrium when delivering data from all source nodes in S to the sink node.

III. Energy Equilibrium Based on Corona Structure

A. Energy Equilibrium of Intra-corona

For $\forall u \in C_i$, let $E_i(u)$ represent the total energy consumption of node u for data gathering during time T, according to energy model given in Section 3.3,

$$E_i(u) = D_{Ti}(u)e_t + D_{Ri}(u)e_r + D_{Fi}(u)e_f \tag{3}$$

where $D_{Ri}(u)$, $D_{Fi}(u)$, $D_{Ti}(u)$ represent the total amount of data received, transmitted, and fused by node u in time T. For all nodes with the same initial energy, the energy equilibrium is obtained in Ci only when

$$E_i(u) = E_i(v),\ \forall \text{ u,v} \in C_i,\ 1 \leq i \leq N \tag{4}$$

Theorem 1: No matter whether data fusion is employed or not, the energy consumption is balanced among nodes in C_i if and only if $D_{Ri}(u) = D_{Ri}(v)$, $\forall$ u,v $\in C_i$.

Proof: Since all nodes in the network have the same data generation rate (L bit/s), the total amount of data generated by each node during time T is TL. From the fusion and slice model given in Section 3.2,

$$D_{Fi}(u) = D_{Ri}(u) + TL \tag{5}$$

$$D_{Ti}(u) = \begin{cases} TL(1\text{-}\sigma)\text{+}D_{Ri}(u), TL \leq D_{Ri}(u) \\ TL\text{+}D_{Ri}(u)(1\text{-}\sigma), TL > D_{Ri}(u) \end{cases} \tag{6}$$

By substituting $D_{Ti}(u)$, $D_{Fi}(u)$ to (7):

$$E_{Ti}(u) = \begin{cases} TL[(1-\sigma)e_t + e_f] + D_{Ri}(e_t + e_r + e_f) \\ TL(e_t + e_f) + D_{Ri}[(1-\sigma)e_t + e_f] \end{cases} \tag{7}$$

Clearly, $E_{Ti}(u)$ is only dependent on $D_{Ri}(u)$ as all the other parameters are the same for the nodes in C_i. When data fusion is not adopted, then $\sigma = 1$, $e_f = 0$, the above conclusion will not be affected. Theorem 1 is proven.

By Theorem 1, the energy equilibrium problem of intra-corona can be simplified as the problem of balancing the amount of data received by nodes within the same corona.

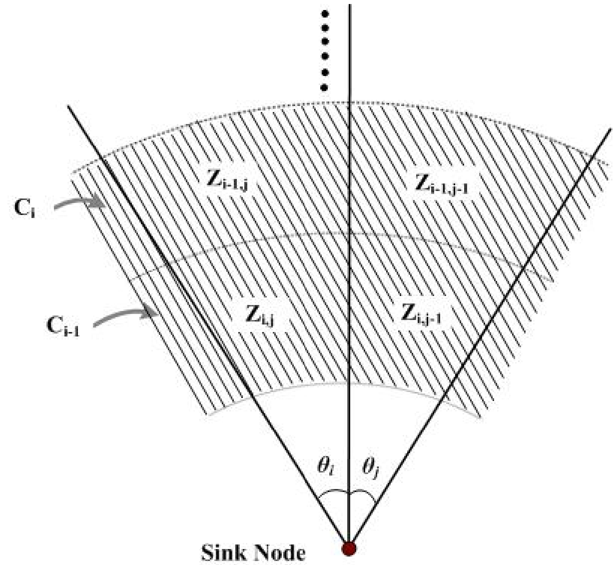

Fig. 1. Example of zone division

B. Energy Equilibrium of Inter-corona

Let S_i and E_i denote the area of corona C_i and the energy consumed per unit time by all the nodes in corona C_i. We assume that the sending power is equal for inter-coronas transmission and the energy consumption for 1 bit is e_t . The energy equilibrium of inter-coronas is obtained when:

$$\frac{S_i \rho E_0}{E_i} = \frac{S_j \rho E_0}{E_j} \quad \forall i, j \in (1, 2, \ldots, N) \tag{8}$$

where E_0 is the initial energy of each node. The amount of data increases gradually from outer corona to inner corona during data gathering. We set a restriction condition: the receiving data of corona $\{C_i|\ 1 \leq i \leq R\}$ is more than the data generated by themselves. [6] has proved that the energy equilibrium of the whole network is unrealizable under the non-uniform distribution. For the node uniform distribution strategy, we can draw the same conclusion.

Theorem 2: When the nodes are uniformly distributed, the energy equilibrium in all the coronas is unrealizable.

Proof: The nodes in corona C_N only needs to transmit data generated by themselves, so the energy consumed per unit time by coronas C_N is expressed as:

$$E_N = S_N \rho L e_t \tag{9}$$

The energy consumed per unit time by coronas C_{N-1} is

$$E_{N-1} = S_N \rho L[(1-\sigma)e_t + e_r + e_f] + S_{N-1}\rho L(e_t + e_f) \tag{10}$$

To achieve energy equilibrium between corona C_N and C_{N-1}.

$$\frac{S_{N-1}\rho L}{E_{N-1}} = \frac{S_N \rho L}{E_N} \Rightarrow S_{N-1}E_N = S_N E_{N-1} \tag{11}$$

we substitute (10) and (11) to (12), and then simplify:

$$S_{N-1}^2 \rho L[(1-\sigma)e_t + e_r + e_f] + S_N S_{N-1}(e_t + e_f) = 0 \tag{12}$$

It is impossible for (13) hold. Hence, theorem 2 is proved.

Although it is unrealizable for energy equilibrium in the whole network, we can prove that it can be achieved in the corona $C_i|\ 1 \le i \le N-1$.

Theorem 3: If the corona area increases in geometric progression from C_{N-1} to C_1 with a common ratio q, $Si\rho L \le D_T(i+1)$, $S_N = S_{N-1}/(q-1)$, $D_T(N) = S_N\rho L(1-\sigma)$, then corona C_1, C_2, ..., C_{N-1} can realize the energy equilibrium of inter-coronas.

Proof: If the coronas can achieve the energy equilibrium of inter-coronas, then:

$$\begin{aligned} \frac{S_i\rho L E_0}{E_i} &= \frac{S_{i+1}\rho L E_0}{E_{i+1}}, 1 \le i \le N-2 \\ &\Rightarrow \quad S_i E_{i+1} = S_{i+1} E_i \end{aligned} \tag{13}$$

When $D_T(N) = S_N\rho L(1-\sigma)$.

$$\begin{aligned} E_i &= \sum_{k=i}^{N} S_k\rho L(1-\sigma)(e_t+e_s) \\ &+ [S_i\rho L + \sum_{k=i+1}^{N} S_k\rho L(1-\sigma)e_f] \\ &+ \sum_{k=i+1}^{N} S_k\rho L(1-\sigma)e_r \\ &= S_i\rho L[(1-\sigma)e_r + e_f] \\ &+ \sum_{k=i+1}^{N} S_k\rho L(1-\sigma)(e_t+e_r+e_f) \end{aligned} \tag{14}$$

After simplification, then we get:

$$\begin{aligned} \frac{S_i}{S_{i+1}} &= \frac{\sum_{k=i+1}^{N} S_k}{\sum_{k=i+2}^{N} S_k} = \frac{S - \sum_{k=1}^{i} S_k + S_i}{S - \sum_{k=1}^{i+1} S_k + S_{i+1}} \\ &= \frac{S - \sum_{k=1}^{i-1} S_k}{S - \sum_{k=1}^{i} S_k} = \frac{S_{i-1}}{S_i}, 2 \le i \le N-2 \end{aligned} \tag{15}$$

Let $\frac{S_i}{S_{i+1}} = \frac{S_{i-1}}{S_i} = q > 1$, then:

$$\frac{S_1}{S_2} = \frac{S_2}{S_3} = \ldots = \frac{S_{N-2}}{S_{N1}} = q \tag{16}$$

Additionally,

$$\begin{aligned} \frac{S_i}{S_{i+1}} &= \frac{\sum_{k=i+1}^{N} S_k}{\sum_{k=i+2}^{N} S_k} = \frac{\frac{S_{N-1}(1-q^{N-1-i})}{1-q} + S_N}{\frac{S_{N-1}(1-q^{N-2-i})}{1-q} + S_N} = q \\ &\Rightarrow \quad S_N = S_{N-1}/(q-1) \end{aligned} \tag{17}$$

The fundamental condition for energy equilibrium is that the area of corona increases in geometric progression with a common ratio of q from C_{N-1} to C_1 . In that case, Theorem 3 is proved.

According to Theorem 3, the data sent by corona C_N to C_{N-1} should be $S_N\rho L(1-\sigma)$. The data amount can be reduced to $S_N\rho L(1-\sigma)$ by data fusion of inter-coronas or data compression. Moreover, it must meet $S_i\rho L \le D_T(i+1)$ to guarantee the energy equilibrium of inter-coronas. This condition limits the value range of q.

Lemma 1: To achieve energy equilibrium, the value of q is in the range of $(1, 2-\sigma]$.

Proof: The precondition for Theorem 3 is $S_i\rho L \le D_T(i+1)$, then

$$\begin{aligned} D_T(i+1) &= \sum_{k=i+1}^{N} S_k\rho L(1-\sigma) \ge S_i\rho L \\ 1 &\le i \le N-1 \end{aligned} \tag{18}$$

For $S_N = S_{N-1}/(q-1)$,$D_T(N) = S_N(1-\sigma)\rho L \ge S_{N-1}\rho L$,

$$\frac{S_{N-1}(1-\sigma)}{q-1} \ge S_{N-1} \Rightarrow q \le 2-\sigma \tag{19}$$

For $D_T(N-1) = (S_N + S_{N-1})(1-\sigma)\rho L \ge S_{N-2}\rho L$,

$$(1-\sigma)[S_N + S_N(q-1)] \ge S_N(q-1)q \Rightarrow q \le 2-\sigma \tag{20}$$

For $D_T(i+1) = \sum_{k=i+1}^{N} S_k\rho L(1-\sigma) \ge S_i\rho L$,

$$\begin{aligned} &S_N(1-\sigma) + S_N(q-1)(1-\sigma)(1+q+\ldots+q^{N-i-2}) \\ &\ge S_N(q-1)q^{N-i-1} \\ &\Rightarrow (1-\sigma) + (q-1)(1-\sigma)\frac{1-q^{N-i-1}}{1-q} \ge (q-1) \\ &\Rightarrow q \le 2-\sigma \end{aligned} \tag{21}$$

Above all, the value of q is in the range of $(1, 2-\sigma]$, Lemma 4 is proven.

IV. Energy Equilibrium Routing

In this section, we design an energy equilibrium routing based on the above analysis, which is aimed at achieving the maximal energy equilibrium with uniform node distribution under the corona structure. The first task of EER is to divide a given circle area A into N coronas.

By Theorem 1, all the nodes in the same corona should receive equal amounts of data to realize the energy equilibrium of intra-coronas. To meet this condition, we divide each coronas into zones within the same area. As shown in Figure 1, the source corona C_{i+1} is divided into zone $Z_{i+1,1}$, $Z_{i+1,2}$, ..., $Z_{i+1,m}$, and the destination corona C_i is divided into m zone $Z_{i,1}$, $Z_{i,2}$, ..., $Z_{i,m}$. During data gathering, all the nodes in $Z_{i+1,j}$ need to equally allocate their data to all the nodes in $Z_{i,j}$.

To save energy, each node has two states: active state and sleeping state. During the active state, the sensor nodes can fulfill the transmission, reception and fusion tasks. During the sleeping state, the sensor nodes turn off their radio model and need not do any other task.

As shown in Figure 2, the operation of EER is divided into several rounds. Each round can be further divided into N time-slots t_0, t_1, t_2, ..., t_{N-1} for data gathering and a common time T_s for all the nodes in the sleeping state.

Nodes in corona C_{N-i} and C_{N-i-1} wake up to fulfill their task in time-slot t_i.

In equation (3), we described the final optimization objective, which is to realize the maximal energy equilibrium for a given area. Based on the above analysis, the following lemma

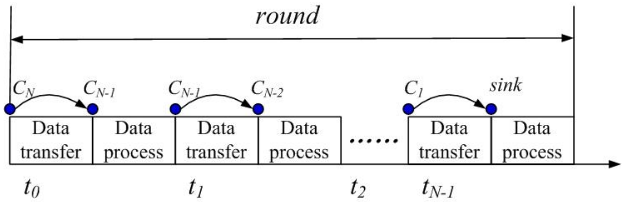

Fig. 2. Data gathering in different time-slots

3 gives the optimization result.

Lemma 3: The maximum ratio of the energy equilibrium area to the total area is $P_e = 1 - q^{1-N}$.

Proof: According to Theorem 3, the minimum area of unrealizable energy equilibrium equals to the area of corona S_N?then:

$$P_e = \sum_{i=1}^{N-1} S_i / \sum_{i=1}^{N} S_i = \sum_{i=1}^{N-1} S_i / (S_N + \sum_{i=1}^{N} S_i) \quad (22)$$
$$= \frac{\frac{(q-1)S_N(1-q^{N-1})}{1-q}}{\frac{(q-1)S_N(1-q_{N-1})}{1-q} + S_N}$$

After simplification $Pe = 1 - q^{1-N}$, Lemma 3 is proven. We use r_{ti} to represent the minimum communication distance of the nodes in corona C_i. r_{ti} should be larger at least than the sum of the width of coronas C_i and C_{i-1}.

V. Simulation Results and Analysis

In all simulations, the sensor nodes are uniformly deployed in a circular area. There is only one sink node which is located at the center of the circle. Except for the sink node without energy limitation, all sensor nodes have the same initial energy 50J. The network lifetime is defined as the time elapsed until the first node in the network runs out of energy. For the radio model, the parameters are set as follows: $E_{elec} = 50nJ/bit$, $\varepsilon_{amp} = 0.0013pJ/bit/m^4$. All simulations are based on a collision-free MAC protocol without data loss. The running time of each round during data gathering is 60 seconds. In each round, each sensor node generates 500 bits of data. The correlation coefficient between the local data and the receiving data changes from 0.1 to 0.5.

We deployed the sensor nodes uniformly in a circular area with $3 \times 10^5 m^2$ and divided the area into 6 coronas. Figure 3 shows the average remaining energy of nodes in each corona when the network lifetime ends. We observe that the nodes in the outermost corona C_6 have more remaining energy when the network lifetime ends no matter whether q is set at 1.2, 1.4 or 1.6 or not. In contrast, the average remaining energy of the nodes belonging to cornoas from C_5 to C_1 is less than 0.04 J, which means that these nodes almost exhaust their energy simultaneously.

We set the energy consumption of data fusion at $15nJ/bit$ and the correlation coefficient changes from 0 to 0.35. Figure 4 illustrates the simulation results of the remaining energy ratio with different correlation coefficients and q. It can be observed that the remaining energy ratio decreases with increasing q for the same correlation coefficient. When q is the same, the remaining energy ratio decreases with an increase in the correlation coefficient. Nonetheless, the remaining energy ratios under different parameters in the simulations are all quite low.

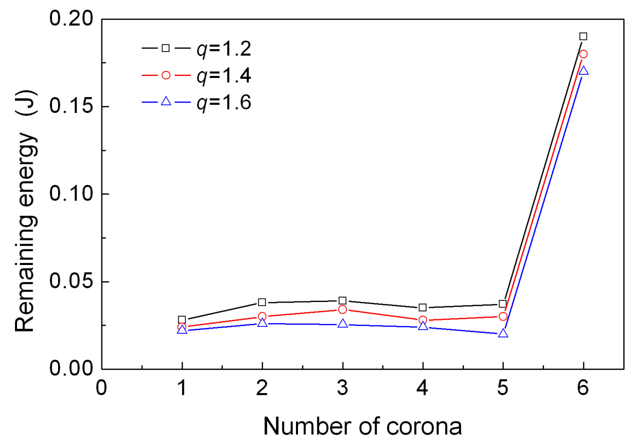

Fig. 3. Remaining energy of different coronas

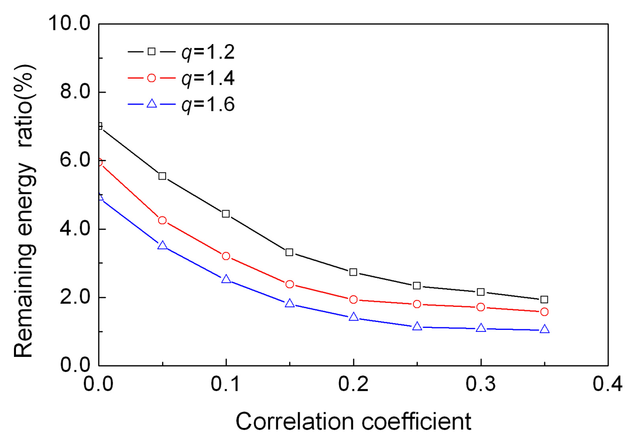

Fig. 4. Effect of q on remaining energy ratio with increasing correlation coefficient

VI. Conclusion

In this paper, we theoretically explore the existence of energy hole that occurs when the nodes are uniformly distributed in WSNs. Then, we formulate the energy equilibrium problem by optimal allocating the corona structure. We found that the energy equilibrium can be realized except for the outmost corona when the corona area increased in geometric progression from the outer corona to the inner corona. By giving the increasing ratio between the areas of neighboring corona C_{i+1} to C_i, we present the solutions for energy equilibrium of intra-corona and inter-coronas. Based on our analysis, an energy equilibrium routing named as EER is proposed. Extensive simulations are performed to validate our analysis. Our simulation results show that EER can guarantee the energy equilibrium.

VII. ACKNOWLEDGEMENT

This work is partially supported by Natural Science Foundation of China under Grant No. 61070181.

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